

Giant Gravitons in type IIA PP-wave Background

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We examine giant gravitons with a worldvolume magnetic flux q in type IIA pp-wave background and find that they can move away from the origin along x^4 direction in target space satisfying $Rx^4 = -q$. This nontrivial relation can be regarded as a complementary relation of the giant graviton on IIA pp-wave and is shown to be connected to the spacetime uncertainty principle. The giant graviton is also investigated in a system of N D0-branes as a fuzzy sphere solution. It is observed that x^4 enters into the fuzzy algebra as a deformation parameter. Such a background dependent Myers effect guarantees that we again get the crucial relation of our giant graviton. In the paper, we also find a BIon configuration on the giant graviton in this background.

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I. INTRODUCTION

Giant gravitons have attracted substantial attention since the pioneering paper [1] of McGreevy, Susskind and Toumbas. They found that a moving particle in $AdS \times S$ background can blow up into a spherical brane (giant graviton) with the increasing angular momentum, and the stringy exclusion principle [2] can be naturally understood as the fact that no particle is bigger than the sphere that contains it. Later in [3] this remarkable mechanism is shown to be another beautiful manifestation of the spacetime uncertainty principle [4] in string theory as well as M-theory. Some important aspects of giant gravitons have been studied in [5]-[11]. On the other hand, pp-wave as the Penrose limit [12] of $AdS \times S$ geometry provides nontrivial but viable backgrounds to test AdS/CFT correspondence and to investigate string theories and various brane configurations [13]-[35]. So it is interesting to investigate giant gravitons in such backgrounds.

Actually there has been much progress along this direction. It was found that the giant gravitons on pp-wave generally have two different guises. If the giant graviton wraps the boosted circle of AdS background, the Penrose limit gives a rotating lightcone brane. In string theory it can be analyzed using the lightcone worldsheet theory. The other type is the static spherical brane, if the giant graviton is not wrapping the boosted circle before we take the limit. These giant gravitons can be examined in D-brane effective action or matrix model. In [36] these two kinds of giant gravitons on type IIB pp-wave have been investigated using the Born-Infeld action and the string worldsheet theory. It was found that giant gravitons with magnetic flux can grow up from multiple D-strings. But there is a suspicion that these D-strings will eventually decay into a giant graviton. In M-theory, the giant gravitons on pp-wave are studied in BMN matrix model [13]. They appear as concentric fuzzy spheres with radius proportional to the lightcone momentum.

In this paper we focus on some configurations of giant graviton in a type IIA pp-wave background, which is the KK reduction of the maximally supersymmetric pp-wave of M-theory. We expect some new features to arise in this background due to the compactification. As a matter of fact, we find that the giant graviton on this pp-wave can have an extra parameter x^4 as the location in transverse space. When a magnetic flux on worldvolume is turned on, the giant graviton will move away from the origin along x^4 while still preserving all the supersymmetries. It is also found that the radius and the location of this giant graviton are subject to a constraint $Rx^4 = -q$. As will be elaborated later, this relation is quite nontrivial since it is the core relation to understand the giant graviton and the spacetime uncertainty principle. Furthermore, since the magnetic flux on giant graviton induces a D0-brane charge, we consider our giant graviton to be composed of multi D0-branes and can be analyzed in the non-abelian Born-Infeld action. We find the fuzzy sphere configuration perfectly matches our giant graviton, and the coordinate x^4 appears in the fuzzy algebra as a deformation parameter. It is also noticed that our giant graviton admits a stringy description in the matrix string theory on pp-wave. On the other hand, recently it was found in [38] that spikes can grow on a dielectric

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brane when an electric flux on the worldvolume is turned on. But the background considered there is not a real supergravity solution. So we wonder whether such a configuration is possible in pp-wave background. In this paper we do find such a 1/8 BPS state exists as a BIon configuration on a giant graviton.

The organization of this paper is as follows. In Section 2 we examine the giant graviton in type IIA pp-wave background using the abelian Born-Infeld action. The BPS solution with a magnetic flux is obtained. The relation to the giant graviton of M-theory is addressed, and the connection to the spacetime uncertainty relation is also elucidated. In Section 3 we consider multiple D0-branes blowing up into fuzzy spheres on the pp-wave as a microscopic description of our giant graviton. We will also have a brief discussion on how the giant graviton arise in matrix string theory. The last section is devoted to the giant graviton with electric flux in worldvolume. The BPS equation is written down for the supersymmetric configuration. Solutions to this equation are discussed and explanations are given. In the appendix, we summarize some useful facts and conventions about the pp-wave background considered in this paper.

II. GIANT GRAVITONS ON IIA PP-WAVE

In this part we consider the giant graviton in the following type IIA pp-wave background

$$ds^2 = -2dx^- dx^+ - \left(\frac{\mu^2}{9} \sum_{i=1}^4 (x^i)^2 + \frac{\mu^2}{36} \sum_{i'=5}^8 (x^{i'})^2\right) (dx^+)^2 + \sum_{I=1}^8 (dx^I)^2$$

$$F_{+123} = \mu, \quad F_{+4} = -\frac{\mu}{3}, \quad (1)$$

by analyzing a spherical D2-brane from the Born-Infeld action. More details about the background can be found in the appendix. The low energy effective action of a D2-brane in general type IIA backgrounds includes the Born-Infeld action and the Chern-Simons terms, which reads as

$$S_2 = -T_2 \int d^3\sigma (e^{-\phi} \sqrt{-\det(P[G+B]_{ab} + \lambda F_{ab})}) + \mu_2 \int P[\sum C^{(n)} e^B] e^{\lambda F}, \quad (2)$$

where $T_2 = 2\pi/(2\pi l_s)^3 g_s$ is the D2-brane tension and $\lambda = 2\pi l_s^2$. As usual $P[\dots]$ is used to clarify the pullbacks, and the potential $C^{(n)}$ of the RR field is defined by $F^{(n+1)} = dC^{(n)}$. μ_2 is the RR charge of the D2-brane and supersymmetry requires that $\mu_2 = \pm T_2$ corresponding to branes or antibranes. We should choose an anti-brane if we assume μ to be positive in the following. For the type IIA pp-wave background of our interest the action is reduced to

$$S_2 = -T_2 \int d^3\sigma (\sqrt{-\det(P[G]_{ab} + \lambda F_{ab})}) + \lambda C_0 F_{12} + C_{012} \quad (3)$$

The spherical D2-brane considered here has an embedding as

$$x^+ = t, \quad x^1 = R \sin \theta \cos \phi, \quad x^2 = R \sin \theta \sin \phi, \quad x^3 = R \cos \theta, \quad (4)$$

where $\{t, \theta, \phi\}$ are chosen as the worldvolume coordinates. We also switch on a worldvolume magnetic flux

$$F_{\theta\phi} = \lambda^{-1} q \sin \theta. \quad (5)$$

Quantization of the magnetic flux requires

$$N = \frac{1}{2\pi} \int d\theta d\phi F_{\theta\phi} = 2\lambda^{-1} q, \quad (6)$$

where N is an integer and is interpreted as the number of D0-branes bound to the D2-brane as can be seen from the coupling of RR 1-form potential. We also have noticed that the transverse scalar x^4 should be nonzero to make the embedding consistent when we have a magnetic flux. Other scalars should be set to zero since the gravity of the background provides a confining potential. In our ansatz the RR potential can be chosen as

$$C_t = \frac{\mu}{3} x^4, \quad C_{t\theta\phi} = -\frac{\mu}{3} R^3 \sin \theta, \quad (7)$$

and one can easily obtain the explicit form of the Lagrangian density

$$\mathcal{L} = -\frac{\mu}{3} T_2 (\sqrt{(R^2 + (x^4)^2)(R^4 + q^2)} + x^4 q - R^3) \sin \theta. \quad (8)$$

Varying x^4 and R gives the equations of motion

$$\begin{aligned} x^4 \sqrt{\frac{R^4 + q^2}{R^2 + (x^4)^2}} + q &= 0, \\ \frac{2R}{3} \sqrt{\frac{R^2 + (x^4)^2}{R^4 + q^2}} + \frac{1}{3R} \sqrt{\frac{R^4 + q^2}{R^2 + (x^4)^2}} - 1 &= 0. \end{aligned} \quad (9)$$

It can be easily checked that these equations are the same as those derived from the full field equations of motion [35] in our ansatz. So our embedding is consistent. The equation (9) can be solved to give

$$Rx^4 = -q. \quad (10)$$

Substituting it back into the Hamiltonian $\mathcal{H} = -\mathcal{L}$, we find that the energy of the configuration exactly equals to zero. Actually this is just the giant graviton solution on IIA pp-wave.

It is interesting to notice that the size of the giant graviton is inversely proportional to its location in x^4 . This relation will be shown to be connected to the spacetime uncertainty principle. For now we can gain some insights by simple analysis. First if x^4 is fixed, then R is proportional to q . In other words we can increase the radius of the giant graviton by adding a magnetic flux to it. This is equivalent to adding D0-branes. Since D0-branes repel each other, the increase of the radius is natural. Second if q is fixed, then R is inversely proportional to x^4 . From the metric of pp-wave we can see that the gravity of the background tends to make a spherical brane collapse. When x^4 increases, this effect becomes stronger and the radius of the giant graviton shrinks. We also notice that the giant graviton with the same sign of q only appear on one side of origin along x^4 . This asymmetry is due to the presence of KK gauge field of the background.

Below we will show explicitly that the solution we obtained above is actually a BPS object using the kappa symmetry projection. For our brane embedding the kappa symmetry projection [39, 40] is

$$\Gamma = -\Delta^{-1}(\tilde{\gamma}_{t\theta\phi} + F_{\theta\phi}\tilde{\gamma}_t\Gamma_{11}). \quad (11)$$

Here $\tilde{\gamma}_i = E_i^A \Gamma_A = \partial_i X^M E_M^A \Gamma_A$ are gamma matrices pulled back on the worldvolume and Γ_A are the gamma matrices in the tangent space of the background. There exists a minus sign in (11) since we consider the case of an anti-brane. Δ is defined as

$$\Delta \equiv \sqrt{-\det(P[G]_{ab} + \mathcal{F}_{ab})}. \quad (12)$$

For the case at hand, we have

$$\begin{aligned} \tilde{\gamma}_t &= \Gamma_+ + \frac{\mu^2}{18}(R^2 + (x^4)^2)\Gamma_-, \\ \tilde{\gamma}_\theta &= R\Gamma_\theta, \\ \tilde{\gamma}_\phi &= R\sin\theta\Gamma_\phi. \end{aligned} \quad (13)$$

Using the above relations, the kappa projection can be written as

$$\Gamma = \Delta^{-1} \sin\theta (\Gamma^- + \frac{\mu^2}{18}(R^2 + (x^4)^2)\Gamma^+) (R^2\Gamma^{\theta\phi} + q\Gamma^{11}). \quad (14)$$

A brane embedding preserves some fractions of the supersymmetry of the background if the Killing spinor η of the background is consistent with

$$\Gamma\eta = \eta, \quad (15)$$

when η is restricted on the worldvolume of the brane. The Killing spinor of the background and the definition of the gamma matrices are given in the appendix. First we use the kappa symmetry projection Γ to act on the kinematical spinor η_1 , and we have

$$\Gamma\eta_1 = -\sqrt{2}\Delta^{-1} \sin\theta \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes (R^2\gamma^{\theta\phi} + q\gamma^9)\tilde{\epsilon}. \quad (16)$$

Since it has a different chirality from η_1 , the 16 kinematical supersymmetries are completely broken. Now we consider the dynamical Killing spinor. The projection gives

$$-\gamma^{R\theta\phi}(R\gamma^R + x^4\gamma^4)(R^2\gamma^{\theta\phi} - q\gamma^9)\epsilon = \Omega\epsilon, \quad (17)$$

$$-\gamma^{R\theta\phi}(R\gamma^R - x^4\gamma^4)(R^2\gamma^{\theta\phi} + q\gamma^9)\epsilon = \Omega\epsilon, \quad (18)$$

which in turn imply the constraints

$$\begin{aligned}(R^3 + qx^4\gamma^{5678})\epsilon &= \Omega\epsilon, \\ (Rx^4 - q\gamma^{5678})\epsilon &= 0,\end{aligned}\tag{19}$$

where $\Omega = \sqrt{(R^2 + (x^4)^2)(R^4 + q^2)}$. These constraints are consistent with $(\gamma^{5678} + 1)\epsilon = 0$ when evaluated on the brane worldvolume. So our spherical configuration preserves all 8 dynamical supersymmetries of the background. Thus it is a 1/4 BPS state. The moving of giant graviton in one direction and still preserving supersymmetry remind us of the phenomenon that a lightcone brane on pp-wave can move along one direction by boosting or adding fluxes on the worldvolume [30, 35]. But the mechanism seems different. We notice that the RR 1-form potential can couple to D0-branes in the form qx^4 like the potential of charges in constant electric field. So D0-branes in this background can feel a force along x^4 and thus can move away from origin until the effects of the gravity increase and balance it.

Now we are ready to see the spherical D2-brane as the giant graviton from M-theory point of view. As the extra dimension x^9 decompactified in the strong coupling limit, the spherical D2-brane can be regarded as a spherical membrane in M-theory pp-wave background. If we further make a coordinate transformation, the background metric is expressed explicitly in the form of maximally supersymmetric M-theory pp-wave (A1), and the static membrane becomes rotating in the plane $X^4 - X^9$ with radius x^4

$$X^4 = x^4 \cos(\frac{\mu}{6}x^+), \quad X^9 = x^4 \cos(\frac{\mu}{6}x^+).\tag{20}$$

Since the maximally supersymmetric M-theory pp-wave is the Penrose limit of $AdS_4 \times S^7$ background, this rotating spherical membrane is identified as the 'dual' giant graviton blown up in AdS_4 and circling in S^7 . From the action of such a rotating giant graviton on M-theory pp-wave, we can easily get the relation $R(x^4)^2 = 2L$, where L is the angular momentum. We first notice that the radius of the giant graviton is proportional to the angular momentum as usual. But if the momentum $p = L/x^4$ of the giant graviton is used, we immediately have $Rx^4 \sim p$, which can be seen as the M-theory counterpart of the relation (10) we found in type IIA theory.

Since the giant graviton on IIA pp-wave has no angular momentum and zero energy, it is not apparent to observe stringy exclusion principle as shown in [1]. However with the essential relation (10), we can argue that the existence of such a giant graviton is actually a manifestation of the spacetime uncertainty relation in M-theory. From M-theory theory point of view, the graviton has an energy

$$E \sim \frac{N}{R_c},\tag{21}$$

where R_c is the radius of the compact direction x^9 and N is the number of D0-branes attached to it. If we consider the giant graviton quantum mechanically and use Heisenberg uncertainty relation, the uncertainty of time is

$$\Delta t \sim \frac{R_c}{N}.\tag{22}$$

And from (10), we know

$$\Delta x^i \Delta x^4 \sim l_s^2 N,\tag{23}$$

where $i = 1, 2, 3$ and $\Delta x^i \sim R$ is assumed. Thus we deduce the relation

$$\Delta x^i \Delta x^4 \Delta t \sim R_c l_s^2 \sim l_p^3\tag{24}$$

This is just the spacetime uncertainty relation in M-theory [3, 4]. So from such a connection, we can say our result about the giant graviton on IIA pp-wave is nontrivial. The relation (10) in some sense reflects the nature of the spacetime geometry.

III. MICROSCOPIC DESCRIPTION OF GIANT GRAVITONS

Since the spherical D2-brane in the previous section has a worldvolume magnetic flux which induces a D0-brane coupling and the background has a 4-form RR field strength, we can consider this D2-brane to be blown-up from N D0-branes as can be seen from the Myers effect [41]. So we investigate a system of N D0-branes in type IIA pp-wave background which we expect to give a microscopic description of the giant graviton found in the previous section.

As usual the transverse scalar $\phi^i \equiv (2\pi\alpha')^{-1}X^i$, where X^i represents a matrix valued coordinate, and we only consider $i = 1, \dots, 4$ in the following since other scalars will not appear in the Wess-Zumino terms. The non-abelian action of N D0-branes for static configurations on type IIA pp-wave is written as

$$S = -T_0 \int dt ST r \sqrt{-G_{00} \det(Q_j^i)} + \mu_0 \int dt \frac{1}{3} \lambda \mu ST r(\phi^4) + \mu_0 \int dt \frac{i}{3} \lambda^2 \mu ST r(\phi^a \phi^b \phi^c) \epsilon_{abc}. \quad (25)$$

Here $Q_j^i = \delta_j^i + i\lambda[\phi^i, \phi^j]$, $G_{00} = -\frac{\mu^2}{9}(\phi^i)^2$, $\lambda \equiv 2\pi\alpha'$ and $a, b, c = 1, \dots, 3$. The symbol $ST r$ as usual means the trace is averaged over all possible orderings of the terms $[\phi^i, \phi^j]$ and ϕ^k appearing inside the trace. In the following we should consider anti-D0-branes by choosing the RR charge $\mu_0 = -T_0$ to adapt to the anti-D2-brane considered in the previous section. Written more explicitly, the action (25) becomes

$$S = -\frac{\mu}{3} T_0 \lambda \int dt ST r \left\{ \sqrt{((\phi^4)^2 + (\phi^a)^2)(1 - \frac{\lambda^2}{2} [\phi^i, \phi^j]^2)} + \phi^4 + i\lambda \phi^a \phi^b \phi^c \epsilon_{abc} \right\}, \quad (26)$$

Since we are interested in the profile of giant graviton blown-up into the transverse space spanned by ϕ^a , the scalar ϕ^4 should commute with ϕ^a . For ϕ^a being an irreducible representation of some algebra, we can choose $\phi^4 = x^4 \mathbb{1}$. Moreover we assume $|x^4|$ large enough compared to the size of giant graviton, i.e., $(\phi^a)^2 \ll (x^4)^2$. The validity of the action requires that the commutators $\lambda[\phi^a, \phi^b] \ll 1$. Expand the square root out in the action and drop out higher order terms of λ , the action can be written as

$$\begin{aligned} V &\simeq \frac{\mu}{3} T_0 \lambda ST r \left(-\frac{1}{2x^4} (\phi^a)^2 + \frac{\lambda^2}{4} x^4 [\phi^a, \phi^b]^2 + i\lambda \phi^a \phi^b \phi^c \epsilon_{abc} \right) \\ &= -\frac{1}{2x^4} \frac{\mu}{3} T_0 \lambda ST r \left(\phi^a - \frac{i}{2} \lambda x^4 [\phi^b, \phi^c] \epsilon_{abc} \right)^2. \end{aligned} \quad (27)$$

Here we have assumed x^4 to be negative. So we can see the energy equal to zero only if

$$[\phi^a, \phi^b] = -\frac{i}{\lambda x^4} \epsilon^{abc} \phi^c. \quad (28)$$

This shows that D0-branes expand into a fuzzy sphere. But interestingly transverse coordinate x^4 enters into the commutator as a noncommutative parameter. If we define $\phi^a = -\frac{J^a}{\lambda x^4}$, then we can see J^a are the generators of the $SU(2)$ algebra $[J^a, J^b] = i\epsilon^{abc} J^c$. The matrix ϕ^a satisfy

$$(\phi^1)^2 + (\phi^2)^2 + (\phi^3)^2 = \frac{1}{\lambda^2 (x^4)^2} C_2(N) \mathbb{1} = \frac{N^2}{4} \frac{1}{\lambda^2 (x^4)^2} \left(1 - \frac{1}{N^2}\right) \mathbb{1}, \quad (29)$$

where $C_2(N) = (N^2 - 1)/4$ is the quadratic Casimir of $SU(2)$ in N-dimensional representation. This shows that the radius of the fuzzy sphere should satisfy

$$R x^4 = -\lambda^{-1} N/2. \quad (30)$$

Turning to original coordinates $X^i = \lambda \phi^i$ gives $R x^4 = -\lambda N/2$. Then if we remember that quantization condition of the magnetic flux requires $N = 2\lambda^{-1} q$, we again arrive at the result $R x^4 = -q$ first exhibited in the abelian action of D2-brane. For a general solution, ϕ^a and ϕ^4 can belong to any N dimensional representation of $SU(2)$ labelled by a partition $\{N_1, \dots, N_k\}$. It corresponds to a set of fuzzy spheres with radii R_i and x_i^4 satisfying $R_i x_i^4 = -\lambda^{-1} N_i/2$.

In the above analysis we have neglected the high order terms of λ in the nonabelian action. To check the validity of the solution in the full action, we substitute the fuzzy algebra

$$[\phi^a, \phi^b] = \frac{2R}{N} i \epsilon^{abc} \phi^c \quad (31)$$

into (26) and perform the trace. The reduced action is

$$S = -\frac{\mu}{3} T_2 \int dt d\theta d\varphi \left\{ \sqrt{(R^2(1 - \frac{1}{N^2}) + (x^4)^2)(R^4(1 - \frac{1}{N^2}) + q^2) + q x^4 - R^3(1 - \frac{1}{N^2})} \right\}. \quad (32)$$

In the large N limit, this action is just the same with (8) for spherical D2-brane in type IIA pp-wave background. So our D0-brane picture do provide a microscopic description of giant graviton. If we further calculate the energy of the

fuzzy sphere satisfying (30), we find a remarkable result that it is exactly zero without large N limit. So the fuzzy sphere is an exact solution of the non-abelian Born-Infeld action (26). The fact that the energy corresponding to N D0-branes is cancelled precisely by the background field indicates that the giant graviton on IIA pp-wave is actually the condensation of these D0-branes.

It is well known that matrix string theory purports to be a nonperturbative formulation of string theory. So our giant gravitons should have a natural description in the matrix string formulism. Actually, such giant gravitons are just the fuzzy spheres found recently in the matrix string theory on pp-wave [37]. Here we have a brief discussion about this solution. Matrix string theory on IIA pp-wave is a (1+1) dimensional Yang-Mills theory [31, 32, 33]. The bosonic terms of the Lagrangian density are as follows [37]

$$\mathcal{L} = Tr \left\{ \frac{1}{2} g_s^2 F_{\tau\sigma}^2 + \frac{1}{2} (D_\tau X^i)^2 - \frac{1}{2} (D_\sigma X^i)^2 - \frac{1}{2} \left(\frac{M}{3}\right)^2 (X^a)^2 - \frac{1}{2} \left(\frac{M}{6}\right)^2 (X^{a''})^2 - \frac{M}{3} g_s X^4 F_{\tau\sigma} - i \frac{M}{3g_s} \epsilon_{abc} X^a X^b X^c \right\}, \quad (33)$$

where M is a constant proportional to μ and α' , g_s is the string coupling constant. The indices have a convention $i = 1 \cdots 8$, $a = 1, 2, 3$ and $a'' = 5 \cdots 8$. The vacua of the theory are fuzzy spheres with a translation in X^4 . Consider the one fuzzy sphere solution

$$X^i = \frac{Mg_s}{3} J^i, \quad X^4 = x^4 \mathbb{1}. \quad (34)$$

We notice that this solution also has an electric flux N which is defined as

$$\frac{1}{2} \oint d\sigma tr E = N, \quad (35)$$

where E is the conjugate variable of the gauge potential A_σ , and can be solved as a zero energy solution

$$E = -\frac{M}{3} g_s X^4. \quad (36)$$

In matrix string theory, the electric flux corresponds to the D0-brane charge of the configuration. So nonzero X^4 indicates that the fuzzy sphere carries D0-branes. On other hand, from (35) and (36) we have

$$x^4 = -\frac{3N}{MN_0 g_s}, \quad (37)$$

where N_0 is the dimension of the representation. Since N is an integer, x^4 can not be chosen continuously. In other words, the fuzzy sphere should locate on discrete positions along X^4 , which means the $U(1)$ group corresponding to X^4 is broken to a discrete subgroup. If (34) and (37) are combined, we instantly get back to the crucial relation (30). This result indicates that matrix string theory also provides a perfect microscopic description of giant gravitons on IIA pp-wave.

IV. GIANT GRAVITONS WITH ELECTRIC FLUX

In [38], it was found that there can be classical stable BIon configuration with S^2 structure as the F-strings bound to dielectric brane. But the background considered in that paper is not real supergravity solution and it is worthy to examine such a profile in a consistent background. In this section we consider the bound state of a giant graviton and n F-strings on IIA pp-wave. This configuration can be analyzed by turning on an electric flux on the worldvolume of a spherical D2-brane. The embedding can be chosen as

$$x^+ = t, \quad x^1 = z, \quad x^2 = R \cos \theta, \quad x^3 = R \sin \theta, \quad F_{tz} = E. \quad (38)$$

The Lagrangian (2) for this embedding can be written as

$$\mathcal{L} = -T_2 \left(\sqrt{\frac{\mu^2}{9} (R^2 + z^2) (R'^2 + 1) R^2 - E^2 R^2} - \frac{\mu}{2} R^2 \right), \quad (39)$$

where $R' \equiv \frac{\partial R}{\partial z}$. We can calculate the Hamiltonian by performing a Legendre transformation

$$\mathcal{H} = T_2 \left(\frac{\mu}{3} \sqrt{(R^2 + D^2) (R^2 + z^2) (R'^2 + 1)} - \frac{\mu}{2} R^2 \right), \quad (40)$$

where $D = \frac{\partial \mathcal{L}}{\partial E}$ is the conjugate variable of the electric field. It is noticed that the above equation can be derived from the membrane action in eleven dimensional pp-wave background with winding number n along the compact direction. And it can be seen from this way that $D = ng_s$ with g_s being the string coupling. Since the equation of motion is quite complicated, we will look for a supersymmetric solution with the aid of the BPS equation.

As in Section 2, we first write down the kappa symmetry projection for this embedding

$$\Gamma = \Delta^{-1}(\tilde{\gamma}_{tz\phi} + E\tilde{\gamma}_\theta\Gamma_{11}), \quad (41)$$

which can be rewritten using gamma matrices in the background as

$$\Gamma = \Delta^{-1}(-R(\Gamma^- + \frac{\mu^2}{18}(R^2 + z^2)\Gamma^+)(\Gamma^{z\theta} + R'\Gamma^{R\theta}) - ER\Gamma^\theta\Gamma^{11}). \quad (42)$$

First the above kappa projection implies that the embedding breaks the 16 kinematical supersymmetry of the background. The projection on the dynamical Killing spinor gives

$$(\frac{\mu}{3}R\tilde{\gamma}\gamma^{zR}\gamma' + ER\gamma^{\theta 9})\epsilon = \Delta\epsilon, \quad (43)$$

$$(\frac{\mu}{3}R\gamma'\gamma^{zR}\tilde{\gamma} + ER\gamma^{\theta 9})\epsilon = -\Delta\epsilon. \quad (44)$$

Here $\tilde{\gamma} \equiv \gamma^z + R'\gamma^R$ and $\gamma' \equiv z\gamma^z + R\gamma^R$. These two equations can be simplified to

$$\gamma^4\epsilon = \pm\epsilon, \quad (45)$$

if the BPS equation

$$RR' + z = \pm \frac{3}{\mu}E \quad (46)$$

is satisfied. So we can see this solution preserves 4 dynamical supersymmetries of the background. When $E = 0$, we have $RR' + z = 0$. This is just the giant graviton we previously discussed. If we employ D instead of E , we have

$$R' = \frac{(\pm D - z)R}{R^2 \pm Dz}. \quad (47)$$

If D is sufficiently small, the D2-brane solution still preserves the spherical structure with a deformation only in the region of small R . When we concentrate on the region $R^2 \ll |Dz|$, the BPS equation can be approximate to

$$R' = \mp \frac{R}{D}. \quad (48)$$

We can easily integrate it to give

$$R = R_0 \exp(\mp \frac{z}{D}), \quad (49)$$

where R_0 is the integration constant. The sign in the exponential of the above equation corresponds to the sign in (45). So the two solutions preserve different factions of the supersymmetry of the background. It is easily noticed that these solutions represent two spikes along $\pm z$ direction, which can be identified with the BI-string carrying a string charge

$$Q_s = \oint d\theta D = nT_f, \quad (50)$$

where n and T_f is the number and tension of the fundamental string. This result indicates that the BIon configuration with spherical structure still exists in pp-wave background. Thus we can explain our configuration as open strings ending on a giant graviton, giving a realization of the Polchinski's D-brane picture.

V. CONCLUSIONS AND DISCUSSIONS

In this paper we have discussed many aspects of the giant graviton on IIA pp-wave. From the Born-Infeld action, we find that such a configuration is quite nontrivial: a giant graviton sitting at different place in x^4 has different size. To be precisely this constraint can be written as $Rx^4 = -\lambda N/2$. We can see that the product of the two transverse coordinates R and x^4 is quantized. If N is fixed, the size of the giant graviton is inversely proportional to x^4 . Thus we regard it as a complementary relation of giant graviton on pp-wave. With this key relation our giant graviton is shown to be a remarkable manifestation of the spacetime uncertainty principle. This result is consistent with previous results on giant gravitons. We also investigate non-abelian action of D0-branes to give a microscopic description of the giant graviton. In this case, the complementary relation is derived naturally from an unusual fuzzy algebra (28), in which we found the coordinate x^4 appears as a deformation parameter. This kind of Myers effect has not been observed before. Further we find the fuzzy sphere is the exact solution of the full non-abelian action without performing large N limit. Thus the giant graviton on IIA pp-wave can be regarded as the condensation of N D0-branes. In the matrix string theory on IIA pp-wave, we also find the same kind of giant gravitons in the form of fuzzy spheres. Since the matrix string theory is argued to be a nonperturbative definition of string theory, it should allow us to study various aspects of the giant graviton on IIA pp-wave. This remains to be a future work. In the last part of this paper, we show a BIon configuration exists on giant graviton in type IIA pp-wave background. And this result generalizes our former result in [38] to a consistent background.

In the end we should point out that all we have discussed in this paper is in the background with positive μ and the giant graviton in it is made up of an anti-brane. But the case with negative μ is also possible. The discussion is parallel if we consider a spherical brane with a positive RR charge as the giant graviton.

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APPENDIX A: TYPE IIA PP-WAVE AND KILLING SPINORS

In this appendix, we warm up the derivation of the type IIA pp-wave from the compactification of the maximally supersymmetric pp-wave in M-theory and give the Killing spinors of this background. This part is mainly based on [34]. The metric we start with is the M-theory pp-wave with 32 supercharges

$$ds^2 = -2dX^+dX^- - \left(\frac{\mu^2}{9} \sum_{i=1}^3 (X^i)^2 + \frac{\mu^2}{36} \sum_{i'=5}^9 (X^{i'})^2\right)(dX^+)^2 + \sum_{I=1}^9 (dX^I)^2, \quad (A1)$$

$$F_{+123} = \mu.$$

This pp-wave background can be obtained as the Penrose limit of $AdS_4 \times S^7$ or $AdS_7 \times S^4$. To be more intuitively this is the local geometry seen by a particle circling with velocity of light in the sphere part of $AdS_4 \times S^7$ or $AdS_7 \times S^4$. It is easily noticed that there are $SO(3) \times SO(6)$ rotational symmetry of X^I and translation symmetry of X^\pm . Besides there are two nontrivial isometries corresponding to the rotations of the X^I and X^- which can be combined to give a spatial isometry and along which we can compactify to obtain a type IIA background. To make the isometry manifest we perform a coordinate redefinition

$$\begin{aligned} X^- &= x^- - \frac{\mu}{6} x^4 x^9, \\ X^4 &= x^4 \cos\left(\frac{\mu}{6} x^+\right) - x^9 \sin\left(\frac{\mu}{6} x^+\right), \\ X^9 &= x^4 \sin\left(\frac{\mu}{6} x^+\right) + x^9 \cos\left(\frac{\mu}{6} x^+\right) \end{aligned} \quad (A2)$$

with other coordinates unchanged. In the new coordinate the metric reads

$$ds^2 = -2dx^-dx^+ - \left(\frac{\mu^2}{9} \sum_{i=1}^4 (x^i)^2 + \frac{\mu^2}{36} \sum_{i'=5}^8 (x^{i'})^2\right)(dx^+)^2 + \sum_{I=1}^8 (dx^I)^2 + \left(dx^9 + \frac{\mu}{3} x^4 dx^+\right)^2. \quad (A3)$$

Now x^9 is a manifest spatial isometry direction. The standard dimension reduction along this direction gives the type IIA background

$$ds^2 = -2dx^- dx^+ - \left(\frac{\mu^2}{9} \sum_{i=1}^4 (x^i)^2 + \frac{\mu^2}{36} \sum_{i'=5}^8 (x^{i'})^2\right) (dx^+)^2 + \sum_{I=1}^8 (dx^I)^2,$$

$$F_{+123} = \mu, \quad F_{+4} = -\frac{\mu}{3}. \quad (\text{A4})$$

For this background, the vielbein can be chosen as

$$e^+ = dx^+, \quad e^- = dx^- + \frac{1}{2} A(x^I) dx^+, \quad e^I = dx^I, \quad (\text{A5})$$

where $A(x^I) \equiv \frac{\mu^2}{9} \sum_{i=1}^4 (x^i)^2 + \frac{\mu^2}{36} \sum_{i'=5}^8 (x^{i'})^2$. The KK gauge field and the RR 3-form potential can be written as

$$A_+ = \frac{\mu}{3} x^4, \quad C_{+ij} = -\frac{\mu}{3} \epsilon_{ijk} x^k. \quad (\text{A6})$$

This IIA background has 24 supercharges since the toroidal compactification along a spatial isometry direction inevitably breaks 8 supercharges [42]. The 24 supersymmetries of type IIA pp-wave background are classified into two classes. 16 of them are non-linearly realized on the string worldsheet and are called kinematical supersymmetry. The other 8, so called, dynamical supersymmetry, are linearly realized and time independent. The kinematical Killing spinor of this type IIA background is

$$\eta_1 = \begin{pmatrix} 0 \\ \tilde{\epsilon} \end{pmatrix}, \quad (\text{A7})$$

where $\tilde{\epsilon} = e^{-\frac{\mu}{6}\gamma^{123}x^+} \tilde{\epsilon}_0^+ + e^{-\frac{\mu}{3}\gamma^{123}x^+} \tilde{\epsilon}_0^-$ and $\tilde{\epsilon}_0^\pm$ are constant spinors satisfying

$$\gamma^{12349} \tilde{\epsilon}_0^\pm = \pm \tilde{\epsilon}_0^\pm. \quad (\text{A8})$$

The dynamical Killing spinor is

$$\eta_2 = \left(1 + \frac{\mu}{6} \Gamma^{123} \Gamma^+ \Gamma^i x^i\right) \begin{pmatrix} \epsilon \\ 0 \end{pmatrix} \quad (\text{A9})$$

Here ϵ should satisfy $(\gamma^{5678} + 1)\epsilon = 0$. In above expressions of Killing spinor, we have chosen the following representation of the 11-dimensional Gamma matrices

$$\begin{aligned} \Gamma^0 &= -i\sigma^2 \otimes \mathbb{1}_{16}, & \Gamma^I &= \sigma^3 \otimes \gamma^I, & \Gamma^9 &= \sigma^1 \otimes \mathbb{1}_{16}, & \Gamma^{11} &= -\sigma^3 \otimes \gamma^9, \\ \Gamma^\pm &= \frac{1}{\sqrt{2}}(\Gamma^0 \pm \Gamma^9), \end{aligned} \quad (\text{A10})$$

where σ 's are Pauli matrices, and $\mathbb{1}_{16}$ is the 16×16 unit matrix. $\gamma^I, I = 1, \dots, 8$ are real symmetric gamma matrices satisfying $Spin(8)$ Clifford algebra $\{\gamma^I, \gamma^J\} = 2\delta^{IJ}$. Γ^{11} and γ^9 are defined as

$$\Gamma^{11} = \Gamma^0 \dots \Gamma^9 = \Gamma^{-+1\dots 8}, \quad \gamma^9 = \gamma^1 \dots \gamma^8. \quad (\text{A11})$$

- [1] J. McGreevy, L. Susskind and N. Toumbas, *Invasion of the Giant Gravitons from Anti-de Sitter Space*, J. High Energy Phys. **06** (2000) 008; hep-th/0003075.
- [2] J. Maldacena and A. Strominger, *AdS3 Black Holes and a Stringy Exclusion Principle*, J. High Energy Phys. **12** (1998) 005; hep-th/9804085.
- [3] M. Li, *Fuzzy Graviton From Uncertain Spacetime*, Phys. Rev. **D 63** (2001) 086002; hep-th/0003173.
- [4] M. Li and T. Yoneya, *Short-Distance Space-Time Structure and Black Holes in String Theory : A Short Review of the Present Status*, Jour. Chaos, Solitons and Fractals (1999); hep-th/9806240.
- [5] M. T. Grisaru, R. C. Myers and O. Tafjord, *SUSY and Goliath*, J. High Energy Phys. **08** (2000) 040; hep-th/0008015.

- [6] A. Hashimoto, S. Hirano and N. Itzhaki, *Large branes in AdS and their field theory dual*, J. High Energy Phys. **08** (2000) 051; hep-th/0008016.
- [7] G. T. Horowitz and V. E. Hubeny, *CFT Description of Small Objects in AdS*, J. High Energy Phys. **10** (2000) 027; hep-th/0009051.
- [8] V. Balasubramanian, M. Berkooz, A. Naqvi and M. J. Strassler, *Giant Gravitons in Conformal Field Theory*, J. High Energy Phys. **04** (2002) 034; hep-th/0107119.
- [9] S. Corley, A. Jevicki and S. Ramgoolam, *Exact Correlators of Giant Gravitons from dual N=4 SYM*, Adv. Theor. Math. Phys. **5** (2002) 809-839; hep-th/0111222.
- [10] B. Janssen and Y. Lozano, *A Microscopical Description of Giant Gravitons*, Nucl. Phys. **B 658** (2003) 281-299; hep-th/0207199.
- [11] B. Janssen, Y. Lozano and D. Rodriguez-Gomez, *A Microscopical Description of Giant Gravitons II: The $AdS_5 \times S^5$ Background*, Nucl. Phys. **B 669** (2003) 363-378; hep-th/0303183.
- [12] R. Penrose, *Any spacetime has a plane wave as a limit*, Differential geometry and relativity, Reidel, Dordrecht, 271, 1976.
- [13] D. Berenstein, J. Maldacena and H. Nastase, *Strings in flat space and pp waves from $N = 4$ Super Yang Mills*, J. High Energy Phys. **04** (2002) 013; hep-th/0202021.
- [14] M. Blau, J. Figueroa-O'Farrill, C. Hull and G. Papadopoulos, *A New Maximally Supersymmetric Background of IIB Superstring Theory*, J. High Energy Phys. **01** (2002) 047; hep-th/0110242.
- [15] R.R.Metsaev, *Type IIB Green-Schwarz superstring in plane wave Ramond-Ramond background*, Phys. Rev. **D 625** (2002) 70-96; hep-th/0112044.
- [16] M. Alishahiha, M. A. Ganjali, A. Ghodsi and S. Parvizi, *On Type IIA String Theory on the PP-wave Background*, Nucl. Phys. **B 661** (2003) 174-190; hep-th/0207037.
- [17] J. Maldacena and L. Maoz, *Strings on pp-waves and massive two dimensional field theories*, J. High Energy Phys. **12** (2002) 046; hep-th/0207284.
- [18] S. Hyun and H. Shin, *Solvable $N = (4, 4)$ Type IIA String Theory in Plane-Wave Background and D-Branes*, Nucl. Phys. **B 654** (2003) 114-134; hep-th/0210158.
- [19] A. Dabholkar and S. Parvizi, *Dp Branes in PP-wave Background*, Nucl. Phys. **B 641** (2002) 223-234; hep-th/0203231.
- [20] A. Kumar, R. R. Nayak and Sanjay, *D-Brane Solutions in pp-wave Background*, Phys. Lett. **B 541** (2002) 183-188; hep-th/0204025.
- [21] D. Bak, *Supersymmetric Branes in the Matrix Model of PP Wave Background*, Phys. Rev. **D 67** (2003) 045017; hep-th/0204033.
- [22] M. Alishahiha and A. Kumar, *D-Brane Solutions from New Isometries of pp-Waves*, Phys. Lett. **B 542** (2002) 130-136; hep-th/0205134;
- [23] S. Hyun and H. Shin, *Branes from matrix theory in pp-wave background*, Phys. Lett. **B 543** (2002) 115. hep-th/0206090.
- [24] P. Bain, K. Peeters and M. Zamaklar, *D-branes in a plane wave from covariant open strings*, Phys. Rev. **D 67** (2003) 066001; hep-th/0208038.
- [25] J.-H. Park, *Supersymmetric objects in the M-theory on a pp-wave*, J. High Energy Phys. **10** (2002) 032; hep-th/0208161.
- [26] Y. Hikida and S. Yamaguchi, *D-branes in PP-Waves and Massive Theories on Worldsheet with Boundary*, J. High Energy Phys. **01** (2003) 072; hep-th/0210262.
- [27] N. Kim and J.-T. Yee, *Supersymmetry and Branes in M-theory Plane-waves*, Phys. Rev. **D 67** (2003) 046004; hep-th/0211029.
- [28] S. Hyun, J. Park and H. Shin, *Covariant Description of D-branes in IIA Plane-Wave Background*, Phys. Lett. **B 559** (2003) 80-88; hep-th/0212343.
- [29] A. Biswas, A. Kumar and K. L. Panigrahi, *p-p' Branes in PP-wave Background*, Phys. Rev. **D 66** (2002) 126002; hep-th/0208042.
- [30] Yoji Michishita, *D-branes in NSNS and RR pp-wave backgrounds and S-duality*, J. High Energy Phys. **10** (2002) 048; hep-th/0206131.
- [31] G. Bonelli, *Matrix Strings in pp-wave backgrounds from deformed Super Yang-Mills Theory*, J. High Energy Phys. **08** (2002) 022; hep-th/0205213.
- [32] K. Sugiyama and K. Yoshida, *Type IIA String and Matrix String on PP-wave*, Nucl. Phys. **B 644** (2002) 128-150; hep-th/0208029.
- [33] G. Bonelli, *Matrix String Models for Exact (2,2) String Theories in R-R Backgrounds*, Nucl. Phys. **B 649** (2003) 130-142; hep-th/0209225.
- [34] S. Hyun and H. Shin, *$N=(4,4)$ Type IIA String Theory on PP-Wave Background*, J. High Energy Phys. **10** (2002) 070; hep-th/0208074.
- [35] K. Skenderis, M. Taylor, *Branes in AdS and pp-wave spacetimes*, J. High Energy Phys. **06** (2002) 025; hep-th/0204054.
- [36] H. Takayanagi and T. Takayanagi, *Notes on Giant Gravitons on PP-waves*, J. High Energy Phys. **12** (2002) 018; hep-th/0209160.
- [37] S. R. Das, J. Michelson, A. D. Shapere, *Fuzzy Spheres in pp Wave Matrix String Theory*; hep-th/0306270.
- [38] Y.-X. Chen and J. Shao, *BIon configuration with fuzzy S^2 structure*, J. High Energy Phys. **07** (2003) 065; hep-th/0306123.
- [39] M. Aganagic, C. Popescu, J. H. Schwarz, *D-Brane Actions with Local Kappa Symmetry*, Phys. Lett. **B 393** (1997) 311-315; hep-th/9610249.
- [40] E. Bergshoeff, P.K. Townsend, *Super D-branes*, Nucl. Phys. **B 490** (1997) 145-162; hep-th/9611173.
- [41] R. C. Myers, *Dielectric-Branes*, J. High Energy Phys. **12** (1999) 022; hep-th/9910053.
- [42] J. Michelson, *(Twisted) Toroidal Compactification of pp-Waves*, Phys. Rev. **D 66** (2002) 066002; hep-th/0203140.